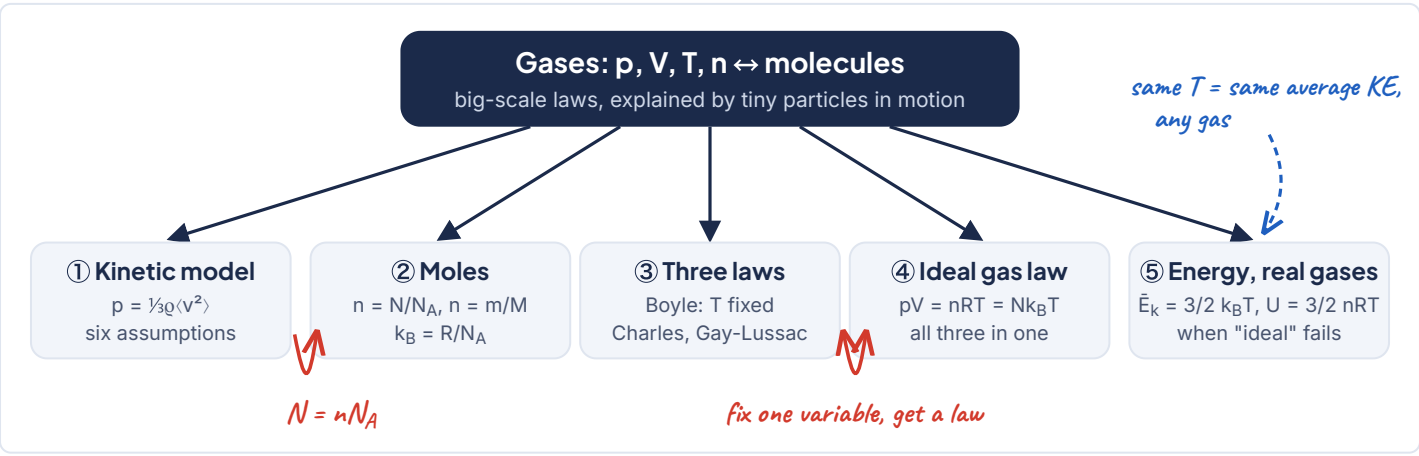
TOPIC B.3 · SL & HL · THEME B THE PARTICULATE NATURE OF MATTER

Gas laws cheatsheet

The kinetic model, moles, Boyle, Charles and Gay-Lussac, the ideal gas law, and when real gases break the rules. Every formula, what it means, a picture and a worked example. Red pen = what I'd tell you in class.

kelvin, kelvin, kelvin. then everything works ↓

~7 teaching hours | Paper 1 & Paper 2 | Paper 3 skill: finding absolute zero | 5 worked examples



1 Pressure and the kinetic model *why gas pushes*

SL/HL · P1 & P2

pressure = millions of tiny bounces

A gas pushes on its container because trillions of molecules **bounce** off the walls. Each bounce changes the molecule's momentum, and the rate of change of momentum is a force. Force per area is the pressure.

FORMULAE

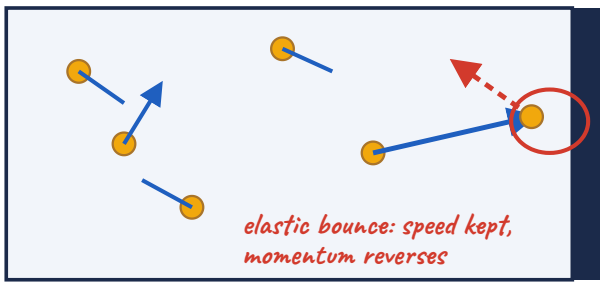
$$p = F / A$$ — *F ⊥ the wall, pascal*

$$\rho = m / V$$

$$p = \tfrac{1}{3}\,\rho\,\langle v^2\rangle$$ — *kinetic model*

$$v_{rms} = \sqrt{\langle v^2\rangle} = \sqrt{(3k_BT / m)}$$

1 Pa = 1 N m^{-2}, 1 atm ≈ 1.0×10^5 Pa. The $\frac{1}{3}$ is because motion is shared over three directions. $\langle v^2\rangle$ is the **mean of the squares**.



push on wall = rate of change of momentum
hotter: faster, harder, more often → higher p
smaller box: more often → higher p

THE SIX MODEL ASSUMPTIONS

1. Many identical molecules, constant random motion.
2. Molecular volume negligible.
3. Collisions perfectly elastic.
4. No intermolecular forces.
5. Collision time negligible.
6. All directions equally likely.

Trap. Pressure comes only from the force **perpendicular** to the surface. In $p = F/A$ use the area the force actually acts on.

WORKED EXAMPLE · NITROGEN AT 0 °C

N_2 molecule mass 4.65×10^{-26} kg, density 1.25 kg m^{-3}, T = 273 K. Find v_{rms} and the pressure.

1. $v_{rms} = \sqrt{(3 \times 1.38\times10^{-23} \times 273 / 4.65\times10^{-26})} =$ **493 m s^{-1}**
2. $p = \frac{1}{3} \times 1.25 \times 493^2 =$ **1.0×10^5 Pa** (1 atm ✓)

Absolute pressure. Gas laws need **absolute** pressure. A gauge reads $p - p_{atm}$, so add ~1.0×10^5 Pa back first.

A gas is mostly empty space: molecules ~10× further apart than in a liquid, so its density is about 1000× smaller.

2 Amount of substance *count in moles*

SL/HL · P1 & P2

MOLES AND THE CONSTANTS

$n = N / N_A$	*N molecules*
$n = m / M$	*M molar mass, kg mol^{-1} or g mol^{-1}*
$k_B = R / N_A$	*$nR = Nk_B$*

$N_A = 6.02\times10^{23}$ mol^{-1} · $R = 8.31$ J mol^{-1} K^{-1} · $k_B = 1.38\times10^{-23}$ J K^{-1}. One mole of a substance has mass M in grams (He 4.0 g, N_2 28 g).

WORKED EXAMPLE · COUNTING ATOMS

How many atoms are in 2.0 g of helium (M = 4.0 g mol^{-1})?

1. n = 2.0 / 4.0 = 0.50 mol
2. $N = 0.50 \times 6.02\times10^{23} =$ **3.0×10^{23}**

equal masses: the lightest molecule has the most molecules. 1 g of H_2 has 14× as many as 1 g of N_2.

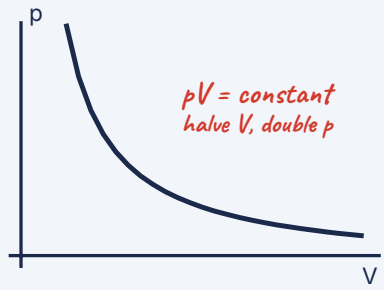
3 The empirical gas laws *fix one, change two*

SL/HL · P1 & P2 · P3

fixed mass of gas, temperature in kelvin

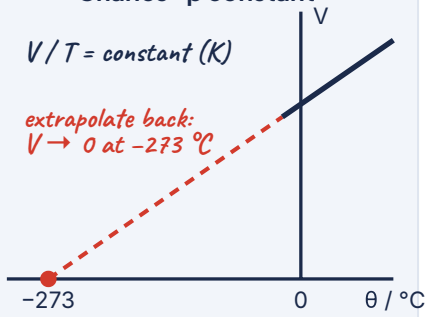
Hold one of p, V, T fixed and the other two are linked. Each law is the ideal gas law with one thing **frozen**.

Boyle · T constant



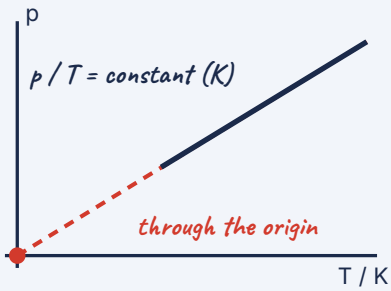
p vs 1/V is a straight line

Charles · p constant



the straight line is in kelvin, not °C

Gay-Lussac · V constant



Paper 3: extrapolate to find absolute zero

THE LAWS AND THE COMBINED FORM

Boyle (T fixed)	$p_1V_1 = p_2V_2$
Charles (p fixed)	$V_1/T_1 = V_2/T_2$
Gay-Lussac (V fixed)	$p_1/T_1 = p_2/T_2$
any change, fixed mass	$p_1V_1/T_1 = p_2V_2/T_2$

WORKED EXAMPLE · COMBINED LAW

A gas at 1.0×10^5 Pa, 20 °C and 500 cm³ is squeezed to 200 cm³ and heated to 60 °C. New pressure?

1. T_1 = 293 K, T_2 = 333 K
2. $p_2 = p_1V_1T_2 / (V_2T_1) = 1.0\times10^5 \times 500 \times 333 / (200 \times 293)$
3. $p_2 =$ **2.8×10^5 Pa**

volumes can stay in cm³ because they cancel

Trap. Never put °C into a gas-law ratio. 0 °C would give zero pressure. Convert: $T = \theta + 273$.

Change	Molecular reason for the effect on p
$V\downarrow$, T const	molecules hit the walls more often
$T\uparrow$, V const	faster: more hits per second, each harder
$T\uparrow$, p const	gas must expand so the hit rate per area falls back

4 The ideal gas law *the big one*

SL/HL · P1 & P2

one equation holds all three laws

An **ideal gas** obeys $pV = nRT$ exactly. Use the form that matches the data you are given: moles or molecules.

TWO FORMS, SAME ANSWER

$$pV = nRT$$ *in booklet*

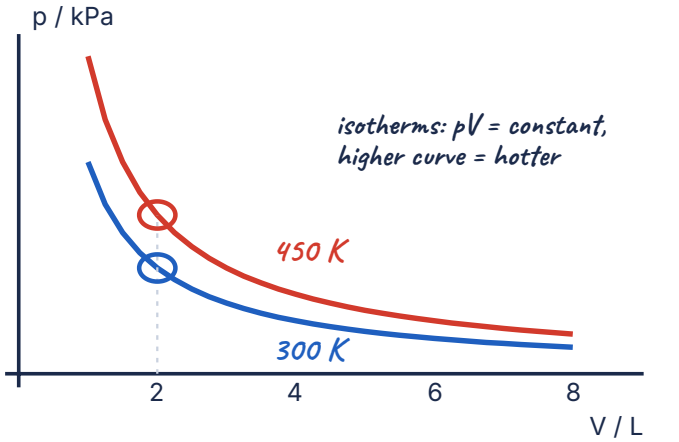
$$pV = Nk_BT$$ *in booklet*

Moles or mass given? Use nRT. A number of molecules? Use Nk_BT. pV/T is constant only while the amount of gas stays fixed.

WORKED EXAMPLE · HOW MUCH GAS?

A 2.0 L flask holds gas at 1.5×10^5 Pa and 27 °C. How many moles and molecules?

1. $V = 2.0\times10^{-3}$ m³, $T = 300$ K
2. $n = pV/RT = 1.5\times10^5 \times 2.0\times10^{-3} / (8.31 \times 300)$ = **0.12 mol**
3. $N = 0.1203 \times 6.02\times10^{23}$ = **7.2×10^{22}**



n = 0.10 mol: at 2 L, p = nRT/V = 125 kPa (300 K), 187 kPa

Reading *p*–*V* shapes. Isotherm = hyperbola. Isobaric = horizontal. Isochoric = vertical. A straight line through the origin on p against $1/V$ means constant T.

Trap. Add gas or let some leak and n changes, so pV/T is **not** constant. The combined law only works for a fixed amount.

5 Kinetic energy and internal energy *temperature = kinetic energy*

SL/HL · P1 & P2

At the same temperature **every** ideal gas has the same **average** kinetic energy per molecule. So the heavier molecules are the slower ones.

ENERGY OF AN IDEAL GAS

$$\bar{E}_k = \tfrac{3}{2}k_BT = \tfrac{3}{2}RT/N_A$$ *in booklet*

$$U = \tfrac{3}{2}Nk_BT = \tfrac{3}{2}nRT$$ *in booklet*

$$v_{rms} = \sqrt{(3k_BT/m)} = \sqrt{(3RT/M)}$$ *M in kg mol⁻¹*

The U formula is for a **monatomic** ideal gas: all KE, no PE (no forces). U depends on T alone, so raising U means raising T. The gradient of a U–T line is $\tfrac{3}{2}nR$.

WORKED EXAMPLE · HEATING AND SPEEDS

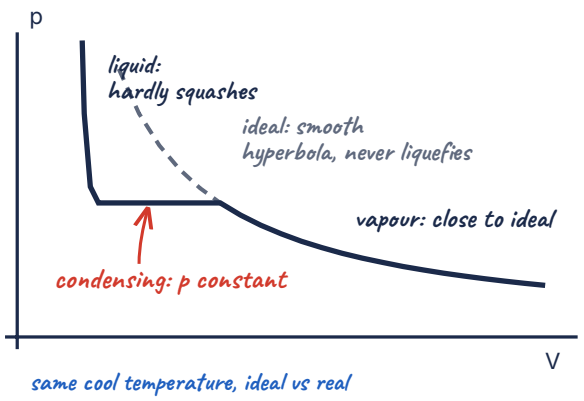
2.0 mol of monatomic gas is warmed by 50 K. Change in U? Compare helium and argon at one temperature.

1. $\Delta U = \tfrac{3}{2} \times 2.0 \times 8.31 \times 50$ = **1.2×10^3 J**
2. $v_{rms} \propto 1/\sqrt{M}$: He (4) against Ar (40) gives $\sqrt{(40/4)}$ = **3.2×** faster for He

Trap. Same T means same average **KE**, not the same speed. Oxygen molecules are slower than helium atoms at one temperature.

6 Real gases against ideal gases *where the model breaks*

SL/HL · P1 & P2



same cool temperature, ideal vs real

	Ideal	Real
molecular volume	negligible	finite
forces between molecules	none	attractive and repulsive
U is	KE only	KE + PE
$Z = pV/nRT$	= 1	≠ 1
condenses?	never	yes

Why it is not ideal

Quote both: molecules have a finite volume *and* there are forces between them. → *two reasons, two marks.*

IDEAL IS A GOOD APPROXIMATION WHEN

Low pressure: molecules far apart, so their volume and forces matter little. **High temperature**: KE ≫ the intermolecular PE, and far from the boiling point. At high p, $Z > 1$ (volume dominates). At low T, $Z < 1$ (attraction lowers p).

Flat part of an isotherm. A flat stretch on a real-gas p–V graph is condensation (a phase change). An ideal gas never shows one.

Before you move on from gas laws

- ✓ Temperature in **kelvin**, pressure **absolute**, volume in m^3 if using R.
- ✓ Fixed amount of gas? Combined law. Amount changes? Go back to $pV = nRT$.
- ✓ Moles or mass given: nRT. Number of molecules: Nk_BT.
- ✓ Know all **six** kinetic-model assumptions.
- ✓ Explain a gas-law result with **molecules**: more hits, harder hits.
- ✓ Same T → same average KE. Heavier → slower.
- ✓ $U = \frac{3}{2}nRT$ is all kinetic: depends on T only.
- ✓ Real gas: finite size **and** forces. Ideal at low p, high T.