PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Voronoi Diagrams

SL 3.5

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FORMULAS YOU MUST KNOW

one of the given points $A, B, C, \ldots$ that the diagram is built around
Site (seed):

the region of all points closer to that site than to any other
Cell:

a boundary between two cells — the perpendicular bisector of the two sites; every point on it is equidistant from them
Edge:

a point where three edges (three cells) meet — equidistant from three sites
Vertex:

$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$
Distance:

$M = \left(\frac{x_1 + x_2}{2}, \frac{y_1 + y_2}{2}\right)$ — every boundary passes through M
Midpoint:

$m_{AB} = \frac{y_B - y_A}{x_B - x_A}$
Gradient of AB:

$m_\perp = -\frac{1}{m_{AB}}$ — flip the fraction and change the sign
Boundary gradient:

$c = y_M - m_\perp x_M$
Intercept:

$y = m_\perp x + c$, equivalently $y - y_M = m_\perp (x - x_M)$
Edge:

the site S_i that minimises $\sqrt{(x_P - x_i)^2 + (y_P - y_i)^2}$
Owner of P:

build the bisectors of $[AB]$ and $[AC]$, then solve them together for (x, y)
Method:

KEY METHODS

- The defining rule is nearest site wins: a location belongs to whichever site is closest. Edges are the two-way ties (equidistant from two sites) and vertices are the three-way ties (equidistant from three).
- For distance, square the gaps, add, then square-root once at the very end. For the midpoint, average the coordinates — add and halve, never subtract.
- Read the two special cases straight off: if $[AB]$ is horizontal ($m_{AB} = 0$) the boundary is vertical, $x = x_M$; if $[AB]$ is vertical the boundary is horizontal, $y = y_M$. The negative-reciprocal rule is only awkward in these two cases.

EXAM TRAPS TO AVOID

- Two classic slips: stopping before the square root, and adding $\Delta x + \Delta y$ instead of squaring each gap. The distance is $\sqrt{(\Delta x)^2 + (\Delta y)^2}$, never $\Delta x + \Delta y$.
- The perpendicular gradient is $-\frac{1}{m}$, not $-m$ and not $\frac{1}{m}$ — you must do both steps. If $m_{AB} = \frac{1}{3}$ then $m_\perp = -3$.
- The line must pass through the midpoint of $[AB]$, not through A or B. Using an endpoint gives a parallel line in the wrong place and loses every follow-on mark.
- Do not eyeball which site "looks" closest on the diagram — a point near an edge can deceive you.

- Point-gradient form $y - y_M = m_{\perp}(x - x_M)$ skips solving for c separately — plug in the midpoint and the perpendicular gradient, then expand.
- To find only the winner, compare squared distances — skip the square roots, since the smallest squared distance is the smallest distance. Take the root only if the question asks for the actual distance.
- Two bisectors are enough — the third passes through the same point automatically. On the GDC, solve the two linear equations as a system.
- Adding a site can only shrink existing cells, never grow them — the new cell is stolen from whatever used to be nearest there.

Store the site coordinates on the GDC and compute every distance.

- The vertex is where the perpendicular bisectors meet — not the medians, and not the triangle's centroid. If your three site-distances come out unequal, the point is wrong.
- After adding D, recompute the nearest site from scratch — do not trust the old winner. Only the cells around D change; distant cells are untouched.
- The optimum must also lie inside the region of interest (e.g. within the map). A point on the boundary of the region can sometimes sit further from every site than any interior vertex — always check the candidates the question allows.