PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Vectors

AHL 3.9–3.14

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FORMULAS YOU MUST KNOW

$\vec{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}$ lists the components along each axis (drop a_3 in 2-D)

Column vector:

$|\vec{a}| = \sqrt{a_1^2 + a_2^2}$

Magnitude (2-D):

$|\vec{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$ — the length of the arrow

Magnitude (3-D):

$\overrightarrow{AB} = \vec{b} - \vec{a}$ — the position vector of B minus that of A

Displacement:

$\hat{a} = \dfrac{1}{|\vec{a}|}\,\vec{a}$ — divide every component by the magnitude

Unit vector:

$|\hat{a}| = 1$ for any non-zero $\vec{a}$

Check:

a vector of length L in the direction of $\vec{a}$ is $L\hat{a}$

Rescale:

$\vec{a} \pm \vec{b}$ adds or subtracts matching components

Add / subtract:

$k\vec{a}$ multiplies every component by k; its length becomes $|k|\,|\vec{a}|$, and $k < 0$ reverses the direction

Scalar multiple:

$\vec{a} + \vec{b}$ is the diagonal of the parallelogram built on $\vec{a}$ and $\vec{b}$

Resultant:

$\vec{a} \cdot \vec{b} = a_1b_1 + a_2b_2 + a_3b_3$

Definition:

$\vec{a} \cdot \vec{b} = |\vec{a}|\,|\vec{b}| \cos\theta$, where θ is the angle between them

Geometric form:

KEY METHODS

- Square each component, add, then square-root — never just add the components. In AI the magnitude is the speed of a velocity vector or the distance of a displacement.
- Find $|\vec{a}|$ first, then divide. A velocity of speed s heading in direction $\vec{d}$ is simply $s\hat{d}$ — normalise the direction, then scale by the speed.
- Line the two columns up and work one row at a time, top to bottom, so no component is dropped or mismatched.
- Two vectors are parallel exactly when one is a scalar multiple of the other: $\vec{a} \parallel \vec{b} \iff \vec{a} = k\vec{b}$ for some

EXAM TRAPS TO AVOID

- The signs of the components never affect $|\vec{a}|$ because squaring removes them — but they entirely decide the direction. Keep a point's position vector and a line's direction vector as separate objects.
- The dot product is a scalar. If your answer still carries components or an $\vec{i}, \vec{j}$, you have made a slip — collapse it to one number.
- A negative dot product means an obtuse angle ($\theta > 90^\circ$). Don't discard the sign — $\cos^{-1}$ returns the correct obtuse angle automatically only if you feed it the signed value.

scalar k. Same k sign means same direction; opposite sign means anti-parallel.

- Line the vectors up component by component — a_1 with b_1, a_2 with b_2 — so you never drop or misalign a term.
- Compute the dot product and both magnitudes separately, then combine — and keep your GDC in degree mode unless radians are asked for.
- The dot product gives $\cos\theta$. HL also has the vector (cross) product, whose magnitude $|\vec{a} \times \vec{b}| = |\vec{a}|\,|\vec{b}|\sin\theta$ gives areas and a perpendicular direction in 3-D — reach for it only when a question asks for an area or a normal vector, not for a plain angle.

- Mind the signs when a component is negative — a dropped minus flips the answer. And $\vec{a} \cdot \vec{b} = 0$ only signals perpendicularity when both vectors are non-zero.
- A point is on the line only if a single t fits all coordinates — matching just one coordinate is not enough. Two lines are parallel when their direction vectors are scalar multiples of each other.
- Distance from the origin is $|\vec{r}(t)|$ — evaluate the position at that time first, then take the magnitude. Taking the magnitude of the velocity by mistake gives the speed, not the distance.
- Crossing paths is not the same as meeting. If the two coordinate equations give different values of t, the paths cross in space but the particles pass that point at different moments — so they never actually meet.