PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Trigonometry

SL 3.1–3.4

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FORMULAS YOU MUST KNOW

$\sin\theta = \dfrac{\text{opp}}{\text{hyp}}$ — so a height along a slope is $\text{hyp} \times \sin\theta$

Sine (SOH):

$\cos\theta = \dfrac{\text{adj}}{\text{hyp}}$ — the horizontal distance is $\text{hyp} \times \cos\theta$

Cosine (CAH):

$\tan\theta = \dfrac{\text{opp}}{\text{adj}}$

Tangent (TOA):

use the inverse, e.g. $\theta = \tan^{-1}\left(\dfrac{\text{opp}}{\text{adj}}\right)$; which two sides you have decides $\sin^{-1}$, $\cos^{-1}$ or $\tan^{-1}$

Find the angle:

$a^2 + b^2 = c^2$, where c is the hypotenuse

Pythagoras:

$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ — each side over the sine of its opposite angle

Sine rule:

$b = a \cdot \dfrac{\sin B}{\sin A}$

Find a side:

flip it: $\dfrac{\sin A}{a} = \dfrac{\sin B}{b}$, so $\sin A = \dfrac{a \sin B}{b}$

Find an angle:

$\sin\theta = \sin(180^\circ - \theta)$, so an acute solution θ has an obtuse partner $180^\circ - \theta$

Two angles, one sine:

keep the obtuse option only if the three angles still sum to less than 180°

Test it:

$a^2 = b^2 + c^2 - 2bc\cos A$, where A is the angle between b and c

Find a side:

$\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$ — the lone side a in the numerator faces the angle A

Find an angle:

KEY METHODS

- Circle the angle first, then ask which side is opposite it. A length along the slope (ladder, cable, string) is the hypotenuse; the height is opposite the ground angle, so use sin; the base distance uses cos.
- Always pair a side with the angle facing it, never an adjacent one. If your answer looks far too big or too small, you have probably flipped the fraction — swap numerator and denominator.
- After the calculator gives the acute angle, always compute 180° minus it and check whether that second triangle is still possible. Many exam

EXAM TRAPS TO AVOID

- The hypotenuse is fixed — it always faces the right angle — but opposite and adjacent swap the moment you switch to the other acute angle. Re-label before every calculation, and keep your GDC in degree mode.
- The sine rule needs a matched side–angle pair. If all you have is two sides and the angle between them, the sine rule cannot start — that is cosine-rule territory (§4).
- The ambiguity only appears when you use the sine rule to find an angle. Finding a side with the sine rule, or using the cosine rule for an angle, is never

questions are worth a mark for spotting both answers.

- When $A = 90^\circ$, $\cos A = 0$ and the $-2bc\cos A$ term vanishes, leaving $a^2 = b^2 + c^2$: the cosine rule generalises Pythagoras, and the extra term is the correction for the angle not being a right angle.
- Only have three sides? Find one angle with the cosine rule first, then feed it into $\frac{1}{2}ab\sin C$. Two sides and their included angle is all this formula ever needs.
- If the angle is given in radians instead, the fraction disappears: arc length $= r\theta$ and sector area $= \frac{1}{2}r^2\theta$, with no degree conversion. Use whichever matches the units in the question.
- Sketch a North line at every vertex and mark the bearings onto it. The interior angle of the triangle is rarely just the difference of the two bearings — the back-bearing at the turn is what you actually need.

ambiguous — the cosine rule returns a single, correctly-signed answer.

- The angle must be the one squeezed between the two known sides. Keep the minus sign attached to $2bc\cos A$, and take the square root only at the very end — squaring, then subtracting, then rooting, in that order.
- The angle must be the included angle between the two sides you multiply, or the formula does not apply. And do not drop the $\frac{1}{2}$: if you compute only $ab\sin C$ you are out by a factor of two.
- Keep the two formulas apart: arc length pairs with the circumference $2\pi r$ (units of length), sector area pairs with πr^2 (units of length squared). Mixing them is the classic slip — a quick unit check catches it.
- Bearings are always three figures and clockwise (072°, not 72° or an anticlockwise angle). Once the triangle is drawn it is usually the cosine rule for the distance, then the sine rule for the returning bearing.