PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Trigonometric Graphs

SL 3.5 · AHL 3.7–3.8

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FORMULAS YOU MUST KNOW

$y = a\sin\big(b(x-c)\big) + d$

Sine form:

$y = a\cos\big(b(x-c)\big) + d$ — the same wave, shifted

Cosine form:

$a = \dfrac{\max - \min}{2}$ — how far the curve swings from the midline

Amplitude a:

$d = \dfrac{\max + \min}{2}$ — the midline $y = d$ the wave oscillates about

Principal axis d:

$\dfrac{360^\circ}{b}$ (degrees) or $\dfrac{2\pi}{b}$ (radians) — the length of one full cycle

Period:

slides the whole wave sideways along the x-axis

Phase shift c:

$a = \dfrac{\max - \min}{2}$ — half the vertical distance between peak and trough

Amplitude:

$d = \dfrac{\max + \min}{2}$ — the average of the peak and trough

Principal axis:

$\text{period} = \dfrac{360^\circ}{b}$ (degrees) or $\dfrac{2\pi}{b}$ (radians)

Period from b:

$b = \dfrac{360^\circ}{\text{period}}$ or $b = \dfrac{2\pi}{\text{period}}$

b from period:

$y_{\text{max}} = d + a$

Maximum:

$y_{\text{min}} = d - a$

Minimum:

KEY METHODS

- The four constants are independent knobs: a stretches the wave vertically, d raises or lowers it, b squeezes it horizontally, and c slides it left or right. Change one and only that feature moves.
- Choose sine or cosine by the starting point. A $+a\cos$ model starts at its maximum at $x = 0$ (since $\cos 0 = 1$); a $-a\cos$ model starts at its minimum; a $\sin$ model starts on the midline, rising. A Ferris wheel boarding at the bottom is a natural $-a\cos$.
- Spot one full cycle in the context — one rotation, one day, one wavelength — and that duration is the period. A Ferris wheel turning once every 4 minutes has period 4, so $b = \dfrac{360^\circ}{4} = 90^\circ$ per minute.

EXAM TRAPS TO AVOID

- Amplitude is half the peak-to-trough distance — forgetting the $\div 2$ is the single most common dropped mark on this topic. It is also always positive: subtract min from max, then halve.
- Period and b are inversely related: a larger b squeezes the graph into a shorter period. If your period comes out bigger when b grows, you have divided the wrong way round.
- Match the GDC mode to the model. A model built with $\dfrac{\pi}{6}$ or $\dfrac{2\pi}{b}$ is in radians; one built with $\dfrac{360^\circ}{b}$ is in degrees. A stray degree setting on a π-model is the number-one error on these questions.
- $\arcsin$ returns only one angle, but a sine equation has two solutions per cycle — the partner is 180° −

- Both $\sin$ and $\cos$ run between -1 and 1, so the model runs between $d-a$ and $d+a$. Add the amplitude to the midline for the peak, subtract it for the trough — differentiating a sinusoid to find its maximum is wasted effort in AI.
- Always in the order d, a, b, c. Nail the midline and amplitude from the max and min, then the period gives b, and only then read the shift c — because c is measured relative to the wave you have already pinned down.
- Go GDC-first: plot both curves, set the window to the given interval, and read every intersection. This finds all solutions without juggling reference angles by hand.
- The two crossings are symmetric about the peak. On the GDC, find both intersection x-values in one period and subtract — the gap is the time spent above the level.

$\arcsin s$ (degrees). Never quote the single principal value when the question asks for solutions "in the interval".

- The answer is the difference of the two times, not the larger value. Reading off only the second crossing (and calling it the duration) is the classic slip here.
- Turn the requirement into $\sin(bt) \geq s$ first, take the arc $180° - 2\arcsin s$, then divide by the full $360°$. Do not divide a duration in hours by the period unless the two are in the same units.