Sequences & Series

FORMULAS YOU MUST KNOW

$u_n = u_1 + (n-1)d$
Arithmetic (add d):

$u_n = u_1 r^{n-1}$
Geometric (times r):

$S_\infty = \dfrac{u_1}{1-r}$ — only when $|r| < 1$ (geometric)
Sum to infinity:

$u_n = u_1 + (n-1)d$
nth term:

$d = u_{n+1} - u_n$ — the same for every consecutive pair
Common difference:

$u_j - u_i = (j-i)\,d$ — solve for d first, then back-substitute for u_1
From two terms:

$S_n = \dfrac{n}{2}(u_1 + u_n)$
Know the last term:

$S_n = \dfrac{n}{2}\big(2u_1 + (n-1)d\big)$
Know d:

$u_n = u_1 r^{n-1}$
nth term:

$r = \dfrac{u_{n+1}}{u_n}$ — the same for every consecutive pair
Common ratio:

$|r| > 1$ grows without bound; $0 < |r| < 1$ shrinks toward 0
Grow or decay:

$S_n = \dfrac{u_1(r^n - 1)}{r-1} = \dfrac{u_1(1-r^n)}{1-r}, \quad r \neq 1$
Sum of n terms:

KEY METHODS

- To classify a sequence, test consecutive terms: a constant difference $u_{n+1} - u_n$ means arithmetic; a constant ratio u_{n+1}/u_n means geometric.
- Given two terms, the number of steps between the i th and jth is $j - i$, not j. Divide the change in value by the change in position to get d in one line.
- Average the first and last term, then multiply by how many terms there are: $S_n = \left(\dfrac{u_1 + u_n}{2}\right) n$. That is all the first formula says.
- The two forms are the same equation — substitute $u_n = u_1 + (n-1)d$ into the first to get the second. Use the top one when you already know u_n, the bottom one when you only know d.
- For "the first term below/above a level", scroll a GDC table of u_n until it crosses — and always check the

EXAM TRAPS TO AVOID

- It takes $(n-1)$ steps to reach the nth term, not n — so both formulas carry the exponent/multiplier $n-1$. Slipping to n is the single biggest source of dropped marks on this topic.
- Count gaps, not terms: reaching the nth term takes $n-1$ jumps of d. Writing $u_n = u_1 + nd$ shifts every answer by one d.
- The exponent is $n-1$, so the first term uses $r^0 = 1$. Writing $u_1 r^n$ shifts every term one place along the sequence.
- The series sum uses r^n (with n = the number of terms), one power higher than the r^{n-1} in the nth-term formula. Mixing the two exponents is a classic error.
- The sum to infinity exists only if $|r| < 1$. If $|r| \geq 1$ the terms do not shrink, the total runs off to infinity,

term just before, since a threshold is an inequality and the answer sits right at the boundary.

- The two forms are identical — use the left one when $r > 1$ and the right one when $|r| < 1$, so the top and bottom stay positive and you avoid sign slips.
- Given u_1 and S_∞, don't re-derive: divide the first term by the sum and subtract from 1 to get $r = 1 - \dfrac{u_1}{S_\infty}$ in one step.

and $\dfrac{u_1}{1-r}$ is meaningless — check r before you quote it.

- n is a whole number and the inequality is strict, so round up (take the ceiling) — never round to nearest. A value of $n > 12.3$ means $n = 13$, and you should confirm S_{12} falls short while S_{13} clears the goal.
- For the long-run limit divide by $1 - r$, never by r. And in the bouncing ball, every bounce is counted twice (up and down) while the first drop counts once — that is why the numerator carries a factor of 2 on the bounces but not on u_1.