Probability

FORMULAS YOU MUST KNOW

$0 \leq P(A) \leq 1$ — and all outcomes in a sample space sum to 1.
Scale:

$P(A) = \dfrac{n(A)}{n(U)}$ — favourable outcomes over total equally likely outcomes.
Theoretical:

$P(A) \approx \dfrac{\text{number of times } A \text{ occurs}}{\text{number of trials}}$ — the relative frequency.
Experimental:

Expected occurrences of A in n trials $=$ $n\,P(A)$.
Expected number:

$P(A') = 1 - P(A)$
Complement:

$P(\text{at least one}) = 1 - P(\text{none})$ — usually far quicker than adding cases.
"At least one":

$P(A \cup B) = P(A) + P(B) - P(A \cap B)$
Addition rule:

If A and B cannot both occur, $P(A \cap B) = 0$, so $P(A \cup B) = P(A) + P(B)$.
Mutually exclusive:

$P(A \cup B) = P(A) + P(B) - P(A)\,P(B)$
Independent union:

$P(A \mid B) = \dfrac{P(A \cap B)}{P(B)}$ — restrict attention to B only.
Conditional:

Rearranged, $P(A \cap B) = P(B)\,P(A \mid B) = P(A)\,P(B \mid A)$.
Multiplication rule:

If A, B are independent, $P(A \mid B) = P(A)$ — the condition tells you nothing new.
Independent case:

KEY METHODS

- As the number of trials grows, the experimental (relative-frequency) probability settles towards the theoretical value — the law of large numbers. A short run of data can look nothing like the true probability.
- Add every category to get the total $n(U)$ — the outcome you want is always included in that total. If probabilities are given as a table, they must sum to 1 , which lets you find any missing value.
- The moment a question says at least one, reach for the complement: find $P(\text{none})$ and subtract. This turns a long "or" calculation into a single step.
- Read the symbols carefully: "$\cup$" (union) means or; "$\cap$" (intersection) means and. In IB, "or" is the

EXAM TRAPS TO AVOID

- Subtract from 1, not from 100, when the probabilities are decimals or fractions. A "percentage" answer still comes from a probability between 0 and 1.
- Subtract the overlap $P(A \cap B)$ exactly once — forget it and the shared part is double-counted, giving a probability that can even exceed 1.
- Independent $\Rightarrow$ multiply for "and"; mutually exclusive $\Rightarrow$ add for "or". Do not mix them up — two events with non-zero probability are almost never both at once.
- The denominator is the given event. Conditioning on "boy" means dividing by the number of boys, not by

inclusive or — it includes the case where both happen.

- To test independence, compare $P(A)\,P(B)$ with the given $P(A \cap B)$: if they are equal the events are independent, if not they are dependent.
- The multiplication rule $P(A \cap B) = P(A)\,P(B \mid A)$ is exactly what you do when you multiply along the branches of a tree diagram (§8) — conditional probability and trees are the same idea.
- Independence straight from the table: $P(\mathrm{Boy})\,P(\mathrm{Bus}) = \frac{18}{30} \cdot \frac{20}{30} = \frac{12}{30} = P(\mathrm{Boy} \cap \mathrm{Bus})$, so here travel mode and gender are independent — the test from §4 in action.

everyone in the sample.

- For a conditional, the denominator is the total of the given row or column — never the grand total N.
- Subtract the "both" count once from each circle before you do anything else, or you will double-count the overlap and the regions will not add up to N.
- Without replacement the second-stage denominator is $n-1$. For "both the same colour" from r red and g green, $P = \dfrac{r}{n} \cdot \dfrac{r-1}{n-1} + \dfrac{g}{n} \cdot \dfrac{g-1}{n-1}$ — reduce both the colour count and the total.