Matrices

FORMULAS YOU MUST KNOW

an $m \times n$ matrix has m rows and n columns; the entry in row i, column j is a_{ij}
Order:

$n \times n$ — only square matrices have a determinant, an inverse and powers
Square matrix:

$I = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ leaves any matrix unchanged: $AI = IA = A$
Identity:

O has every entry 0, so $A + O = A$
Zero matrix:

$(A \pm B)_{ij} = a_{ij} \pm b_{ij}$ — both matrices must have the same order
Add / subtract:

kA multiplies every entry of A by k
Scalar multiple:

$A_{m \times n} B_{n \times p}$ needs the inner dimensions to match; the product is $m \times p$
Conformable:

$(AB)_{ij} = \sum_k a_{ik} b_{kj}$ — row i of A paired term-by-term with column j of B
Entry (i, j):

for a square A: $A^2 = AA,\ A^3 = AAA,\ \ldots,\ A^0 = I$
Powers:

$\det \begin{pmatrix} a & b \\ c & d \end{pmatrix} = ad - bc$
2×2:

enter the matrix and read $\det$ straight from the GDC — no expansion by hand needed in AI
3×3:

$|\det M|$ is the area/volume scale factor; $\det M = 0 \Rightarrow$ the matrix is singular (no inverse)
Meaning:

KEY METHODS

- Two matrices are equal only when they have the same order and every corresponding entry matches — that is how "find the value of x" entry-matching questions are solved.
- Line the two matrices up and work one position at a time, top-left to bottom-right, so entries never get mixed up.
- Run one finger along the row of A and another down the column of B together, multiplying the pairs as you go — that keeps every term matched.
- $\det M = 0$ is the single test for "no inverse / no unique solution". Compute it first — if it is zero, every method that relies on M^{-1} stops here.

EXAM TRAPS TO AVOID

- Order is always (rows × columns), in that order. A 2×3 matrix is a different object from a 3×2 one, and swapping them makes a product undefined.
- You can only add or subtract matrices of the same order. A 2×2 and a 2×3 cannot be combined — the operation is simply undefined, not zero.
- Order matters: in general $AB \neq BA$, so you may never swap the factors. The identity is the one exception, $AI = IA = A$; multiplication is associative and distributive but not commutative.
- Mind the signs in $ad - bc$: a negative times a negative inside bc can flip your answer. Write ad and bc out separately before subtracting.

- Always find $\det M$ first. If it is 0 the matrix is singular — stop, because no inverse exists and $\frac{1}{0}$ is the tell.
- Store A and B as matrices on the GDC and evaluate $A^{-1}B$ in one keystroke — quicker and less error-prone than the simultaneous-equation app, and it works for three unknowns just as easily as two.
- The same idea runs one dimension up: for a 3×3 matrix $|\det M|$ scales volume, and $\det M = 0$ squashes the solid flat — which is exactly why a singular matrix has no inverse.

- In the 2×2 formula you swap a and d but only negate b and c — do not put a minus on a or d, and do not forget to divide the whole matrix by $ad - bc$.
- If $\det A = 0$ there is no unique solution (either none or infinitely many), and the GDC will refuse to invert. That error is information about the system, not a typing mistake — though it is worth re-checking your entries too.
- Areas multiply by $|\det M|$, never by the linear factor alone. A negative determinant is fine — its magnitude still gives the area, and the sign just means the shape's orientation has been reversed.