Loans & Annuities

SL 1.4 · 1.7

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FORMULAS YOU MUST KNOW

a rate of $r\%$ becomes the growth factor $1 + \frac{r}{100}$, applied once per period
Multiplier:

$FV = P\left(1 + \frac{r}{100}\right)^t$ — principal P after t periods at $r\%$ per period
Future value:

$FV = P\left(1 + \frac{r}{100k}\right)^{kt}$ — k compounding periods per year for t years
k times a year:

annually $k = 1$, quarterly $k = 4$, monthly $k = 12$, daily $k = 365$
Common k:

$V = P\left(1 - \frac{r}{100}\right)^t$ — value after t years losing $r\%$ of value per year
Depreciation:

total number of payment periods — for monthly payments, $N = 12 \times \text{years}$
N:

the nominal annual interest rate, entered as a whole number (6, not 0.06)
$I\%$:

present value (principal / lump sum today), the payment each period, and future value ($FV = 0$ to clear a loan or empty a fund)
PV, PMT, FV:

$P/Y = C/Y = 12$ for monthly payments compounded monthly — the solver divides $I\%$ by this for you
P/Y, C/Y:

$FV = D \cdot \frac{(1+i)^n - 1}{i}$ — deposit D each period, rate i per period, n deposits
Future value:

$PV = 0$, $PMT = -D$, enter N and $I\%$, then solve for FV
On the GDC:

$\text{interest} = FV - D \times n$ — the fund minus the plain total paid in
Interest earned:

KEY METHODS

- Turn the rate into a multiplier first: 4% per year means multiply by 1.04 once each year, so t years multiplies by 1.04^t.
- A single amount uses this power formula. A stream of equal payments is an annuity — a geometric series whose value collapses to the present- and future-value formulas in §6–§9.
- Divide the rate by k and multiply the years by k: 6% compounded monthly uses 0.5% per month over $12t$ months.

EXAM TRAPS TO AVOID

- The multiplier is $1 - \frac{r}{100}$, e.g. $\times 0.85$ for 15%. Writing $\times(-0.15)$ or $\times 1.15$ is the classic slip — one makes the value negative, the other makes it grow.
- This is a ratio of two separate logs, not $\ln \frac{A/P}{1 + r/100}$. And because interest is credited only at each period end, round a fractional t up to the next whole period.
- Sign convention. Money you receive is positive; money you pay is negative. A loan has $PV > 0$ (cash in) and $PMT < 0$ (repayments out); a savings

- More frequent compounding earns a little more. The effective annual rate is $\left(1+\frac{r}{100k}\right)^k - 1$ — so a nominal 6% compounded monthly is really about 6.17% a year.
- Subtract the rate from 1: losing 15% a year means multiplying by 0.85 each year, so after t years the value is $P \times 0.85^t$.
- This is a synthesis with Exponents & Logarithms (SL 1.5). If you prefer, skip the algebra and solve $P\left(1+\frac{r}{100}\right)^t = A$ on the GDC with the equation solver or by intersecting graphs.
- The solver converts to the per-period rate $i = \dfrac{I\%}{100 \times (C/Y)}$ automatically, so enter the whole percentage and set $P/Y = C/Y$ correctly — do not divide by 12 yourself as well.

plan has $PV = 0$ with deposits $PMT < 0$. Mixing the signs is the single most common finance error.

- Deposits leave your pocket, so PMT is negative and the resulting FV comes back positive. Set $PV = 0$ when the plan starts from nothing.
- The repayment must exceed the first month's interest $i \times L$, or the balance never falls. Convert to the monthly rate $i = r/1200$ before substituting — don't feed an annual rate into a monthly formula.
- Round the number of payments up, never to nearest: at the floor value a positive balance still remains, so a whole extra (part-)payment is always needed to finish.