PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Kinematics & Projectiles

SL 5.4 · AHL 5.13

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FORMULAS YOU MUST KNOW

$v = \dfrac{ds}{dt}$ — the rate of change (and s–t graph gradient) of displacement

Velocity:

$a = \dfrac{dv}{dt} = \dfrac{d^2 s}{dt^2}$ — differentiate the displacement twice

Acceleration:

$s = \displaystyle\int v\,dt$ — integrate the velocity (add $+C$)

Displacement:

$v = \displaystyle\int a\,dt$ — integrate the acceleration (add $+C$)

Velocity:

$\dfrac{d}{dt}\,(kt^n) = kn\,t^{\,n-1}$ — differentiate each term

Power rule:

differentiate once — $v(t) = s'(t)$

$s \to v$:

differentiate again — $a(t) = v'(t) = s''(t)$

$v \to a$:

$v(t) = 0$ — solve for the time(s)

At rest:

v changes sign at that t — the motion reverses

Changes direction:

solve $a(t) = v'(t) = 0$, then substitute that t back into v

Extreme velocity:

$v(t) = \displaystyle\int a\,dt + C$, with C set by $v(0)$

Velocity from a:

$s(t) = \displaystyle\int v\,dt + C$, with C set by $s(0)$

Displacement from v:

KEY METHODS

- The chain is the topic. Every question is really asking "which way am I moving along $s \to v \to a$?" — differentiate to reach a rate, integrate to reach a total. This is the motion version of differentiation and integration.
- On Paper 2 you can skip the algebra: graph $s(t)$ and read the numerical derivative $\frac{ds}{dt}\big|_{t=p}$ for the velocity, or graph $v(t)$ for the acceleration. Show one line of calculus to bank the method mark.
- Velocity is signed — its sign is the direction of travel. Speed is $|v|$, always ≥ 0. Getting these two confused is the root of most distance-versus-displacement slips (§6).
- "At rest", "stationary", "momentarily at rest", "instantaneously at rest" and "changes direction"

EXAM TRAPS TO AVOID

- Differentiate first, substitute the time second. Plugging the number in before you differentiate collapses the function to a constant, whose derivative is 0.
- "At rest" is not "not accelerating". At $t = 1$ and $t = 3$ above the velocity is zero but the acceleration $a = 6t - 12$ is not — the particle is still slowing or speeding up through the instant it is at rest.
- $v = 0$ means at rest; $a = 0$ means fastest or slowest. Solving the wrong one is the classic mix-up. If a question wants an extreme velocity, set $a = 0$, not $v = 0$.
- The $+C$ is not optional. Forgetting it — or evaluating your target time before applying $v(0)$ — is the single

are all the same cue: set $v = 0$.

- On a v–t graph the extreme velocity is the graph's own turning point, so the GDC's maximum/minimum feature applied to $v(t)$ delivers it in a single step.
- A definite integral $\int_{t_1}^{t_2} v\,dt$ needs no constant and returns the displacement (net change of position) over the interval — which may be positive, negative or zero.
- If the question says "total distance", never integrate v straight through — that returns only the displacement. Solve $v = 0$ first and integrate $|v|$ leg by leg, or key $\int |v|\,dt$ into the GDC.

most common integration error in kinematics. Integrate, apply the initial condition, then substitute.

- Average velocity uses the two displacement endpoints: it is $\dfrac{s(t_2) - s(t_1)}{t_2 - t_1}$, not the value of v at either end, and not the average of the two end velocities.
- For a speed-up/cruise/slow-down journey the area is the whole trapezium, not a triangle or a rectangle. The short parallel side (the "top") is the length of the constant-speed phase — not the maximum speed.
- Only the vertical component decides the time of flight and the maximum height; only the horizontal component carries the range. Never mix the two — resolve the launch velocity before you substitute.