PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Integration

SL 5.5 · AHL 5.11–5.13

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FORMULAS YOU MUST KNOW

$\int ax^n\,dx = \frac{a}{n+1}x^{n+1} + C \quad (n \neq -1)$ — raise the power, then divide by the new power

Reverse power rule:

integrate a polynomial term by term

Sum rule:

$\int a\,dx = ax + C$ — a plain number integrates to ax

Constant term:

$\int_a^b f(x)\,dx = F(b) - F(a)$, where $F'(x) = f(x)$

Definite integral:

$\int_a^b \left(px^2 + qx + r\right) dx = \left[\frac{p}{3}x^3 + \frac{q}{2}x^2 + rx\right]_a^b$

Polynomial:

integrate $\frac{dy}{dx}$ to get $y = F(x) + C$, substitute the given point to solve for C, then evaluate y where required

Method:

$A = \int_a^b f(x)\,dx \quad \text{for } f(x) \geq 0 \text{ on } [a, b]$

Area (above axis):

graph $y = f(x)$, then use the ∫f(x)dx (area/integral) tool between $x = a$ and $x = b$

On the GDC:

$\int_a^b y\,dx \approx \frac{h}{2}\Big(y_0 + y_n + 2\big(y_1 + y_2 + \cdots + y_{n-1}\big)\Big)$

Trapezoidal rule:

$h = \frac{b-a}{n}$ — the equal spacing between readings (n strips, $n+1$ ordinates)

Strip width:

$\int_a^b f(x)\,dx$ — parts below the axis subtract

Net (signed) area:

$\int_a^b \left|f(x)\right|\,dx$ — split at the roots, or integrate the modulus on the GDC

Total area:

KEY METHODS

- Integration and differentiation are inverses, so check any integral by differentiating it — if you don't get back the original integrand, you have slipped somewhere.
- On Paper 2 the GDC evaluates $\int_a^b f\,dx$ directly, so use it to check — but still write down the integral you typed in, because the setup is what earns the method mark.

EXAM TRAPS TO AVOID

- Divide by the new power $n+1$, never the old one, and never drop the $+C$ on an indefinite integral. The rule fails only at $n = -1$, where $\int x^{-1}\,dx = \ln|x| + C$ instead (see §7).
- Keep the polynomial terms lined up: the classic slip is forgetting to divide the x^2 term by 3 or the x term by 2. Substitute the upper limit first, then subtract the lower — reversing them flips the sign of the answer.

- For a region trapped between two curves, the area is $\int_a^b (\text{top} - \text{bottom})\,dx$ — subtract the lower function from the upper, then integrate across the interval where they enclose the region.
- Whether the estimate is too big or too small depends on the curvature: where the graph is concave up (bending upwards) the straight strip-tops lie above the curve, so the rule over-estimates; where it is concave down the rule under-estimates.
- The AI synthesis question is two clean steps: run the regression to turn the table into $R(t)$, then the total over $[t_1, t_2]$ is $\int_{t_1}^{t_2} R\,dt$ — never the single reading $R(t_2)$.

- You cannot skip C. Integrate first, use the given point to solve for C, and only then substitute the target value of x. Substituting the target before you have found C defeats the whole method.
- Area needs integration, not substitution: the height $f(b)$ is never the area. Integrate first. The limits a and b are the x-values that bound the region — often the roots where the curve crosses the axis.
- Only y_0 and y_n are counted once — every value in between is doubled. Miscounting the interior ordinates, or using the number of readings as n instead of the number of strips, are the usual errors.
- If a question asks for the area and the curve dips below the axis, do not simply integrate across the whole interval — that gives the signed (net) value. Split at the roots and add the magnitudes, or evaluate $\int |f(x)|\,dx$ on the GDC.
- Before substituting, check the factor in front is a multiple of the derivative of the inside bracket. If it is out by a constant you can adjust for it; if it is the wrong power of x altogether, the reverse chain rule does not apply.