PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Functions

SL 2.1–2.6 · AHL 2.7

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FORMULAS YOU MUST KNOW

$f : x \mapsto f(x)$ assigns to each input exactly one output
Function:

to find $f(a)$, replace every x in the rule by a, then compute
Evaluate:

$f(x) = ax^3 + bx^2 + cx + d$ — substitute term by term, watching the sign of each
Cubic model:

the inputs x the rule permits — exclude anything that divides by 0 or square-roots a negative, and respect context (e.g. time $t \geq 0$)
Domain:

the outputs $f(x)$ produced as x runs over the domain
Range:

for a linear (monotonic) model the extreme outputs sit at the ends of the interval
On $a \leq x \leq b$:

$(f \circ g)(x) = f\left(g(x)\right)$ — put x into the inner function g first, then its output into f
Composite:

in general $(f \circ g)(x) \neq (g \circ f)(x)$
Order matters:

$f(a) = b \iff f^{-1}(b) = a$, and $f^{-1}\left(f(x)\right) = x$
Reverses f:

if $f(x) = mx + c$ then $f^{-1}(x) = \dfrac{x - c}{m}$ — undo the operations in reverse: subtract c, then divide by m
Linear inverse:

$y = f^{-1}(x)$ is the mirror image of $y = f(x)$ in the line $y = x$
Graphically:

$f(x) = mx + c$ — gradient m is the constant rate of change, intercept c is the value when $x = 0$
Linear model:

KEY METHODS

- On the GDC, type the rule into Y_1 and read $f(a)$ straight from a table or the value tool — faster and safer than hand arithmetic for a cubic or an exponential.
- For a decreasing model the largest input gives the smallest output — so the minimum of the range is at the right-hand end $x = b$, and the maximum at the left end $x = a$.
- For a single value, work inside-out: compute $g(x_0)$ to one number first, then substitute that into f. You never need to build the full composite formula.

EXAM TRAPS TO AVOID

- "One input, one output" is the whole definition. A rule that returns two values for one x — like $y = \pm\sqrt{x}$ — is not a function, so it must pass the vertical-line test.
- Context shrinks the domain. If x counts people or measures time, negative and fractional values are excluded — and the range you quote must match that restricted domain, not the whole real line.
- $f \circ g$ means "f of g", so g acts first. Reading it left-to-right and applying f first is the classic slip — and because order matters, it usually gives a different answer.

- AHL synthesis with differentiation. Reflecting a curve in $y = x$ turns each slope into its reciprocal: if $f(a) = b$ then $\left(f^{-1}\right)'(b) = \dfrac{1}{f'(a)}$, where $a = f^{-1}(b)$. Find a first, differentiate f (not f^{-1}), then take the reciprocal — on a decreasing branch $f'(a) < 0$, so the answer is negative.

- Read the parameters in context: m is a "per unit" rate (dollars per item, °C per minute) and c is the starting amount at $x = 0$. Stating those interpretations with correct units earns the modelling marks.

- Find where the peak is first — $t^* = \dfrac{v}{2u}$ (or $-\frac{b}{2a}$) — then substitute back to get the maximum value. On the GDC, graph the model and use the maximum/minimum tool to confirm both coordinates.

- To find when a target is reached, divide the target by the initial value. If that ratio is a whole power of the base — $2^{t/d} = 2^n \Rightarrow t = nd$ — match exponents exactly; otherwise take $\log$ on the GDC. For a "first exceeds" question, round the answer up.

- f^{-1} exists only when f is one-to-one. A many-to-one rule such as $f(x) = x^2$ must first have its domain restricted to a single branch (here $x \geq 0$) before it can be inverted. And $f^{-1}(x)$ is the inverse, never the reciprocal $\frac{1}{f(x)}$.

- When you evaluate, the answer is mx plus c, not just mx. Dropping the fixed term c — the start-up cost, the initial population — is the single most common careless slip on linear models.

- The vertex has two coordinates. $x = -\frac{b}{2a}$ is where the max/min occurs (the time, the price); the maximum itself is the y-value $f(-\frac{b}{2a})$. Quoting the x when the question wants the y (or vice versa) throws away the answer mark.

- Never drop the added constant. As $x \to \infty$ the $k\,a^x$ (or b^t) term dies away and the model levels off at $y = c$ (or T_a) — not at 0. Reading the asymptote as zero is the signature exponential-model error.