PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Exponents & Logarithms

SL 1.5 · AHL 1.9

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FORMULAS YOU MUST KNOW

$a^m \times a^n = a^{m+n}$ — same base, add the indices
Product:

$\frac{a^m}{a^n} = a^{m-n}$ — same base, subtract the indices
Quotient:

$(a^m)^n = a^{mn}$ — multiply the indices
Power of a power:

$(ab)^n = a^n b^n$ and $\left(\frac{a}{b}\right)^n = \frac{a^n}{b^n}$
Over a product:

$a^0 = 1$ for any $a \neq 0$
Zero index:

$a^{-n} = \frac{1}{a^n}$ — a negative index means reciprocal, giving a positive fraction
Negative:

$a^{1/n} = \sqrt[n]{a}$ — the n-th root
Unit fraction:

$a^{m/n} = \left(\sqrt[n]{a}\right)^m = \sqrt[n]{a^m}$ — root first, then power
General fraction:

$P(t) = A\,b^t$ — initial value A (at $t = 0$), growth/decay factor b
Exponential model:

$b > 1$ grows, $0 < b < 1$ decays; growth rate $= b - 1$, decay rate $= 1 - b$
Grow or decay:

$b^{ax+c} = b^k \;\Rightarrow\; ax + c = k$ — equal bases force equal indices
Same-base equation:

$x = \log_b a \iff b^x = a$, valid for $b > 0,\ b \neq 1,\ a > 0$
Definition:

KEY METHODS

- When the bases already match you never need the calculator: add the top indices and subtract the bottom one, e.g. $\frac{2^7 \times 2^3}{2^4} = 2^{7+3-4} = 2^6 = 64.$
- Read the fraction as "denominator = root, numerator = power", and take the root first to keep the numbers small: $27^{2/3} = \left(\sqrt[3]{27}\right)^2 = 3^2 = 9.$
- Type the model in one line on the GDC as `A*b^t` and use the power key: a factor $b = 1.05$ is 5% growth per step, $b = 0.92$ is 8% decay. Never multiply the base by t.
- Read every logarithm aloud as a power: $\log_2 32 = 5$ because $2^5 = 32$, and $\log_{10} 1000 = 3$ because $10^3 = 1000$. Your GDC evaluates any $\log$ or $\ln$ directly.

EXAM TRAPS TO AVOID

- These laws only apply to a common base. $2^3 \times 5^2$ cannot be merged into a single power, and $a^m + a^n$ never simplifies by adding indices — the laws are about multiplying, not adding.
- A negative index is not a negative number: $2^{-3} = \frac{1}{8}$, never -8. Flip the base to make the index positive, then evaluate the fraction.
- Same-base equating only works once both sides are a power of the same base. Rewrite first: $8^x = 2^{x+5} \Rightarrow (2^3)^x = 2^{x+5} \Rightarrow 3x = x + 5 \Rightarrow x = 2.5$. If no common base exists, take logs (§7).
- The laws act on products and quotients, never on sums: $\log(x + y) \neq \log x + \log y$. And a quotient

- Because the argument a must be positive, the $\log$ of 0 or of a negative number is undefined — a solution that forces a negative argument must be rejected.
- Combine into a single log, then ask "what power of the base is the argument?": $\log_5 100 - \log_5 4 = \log_5 25 = 2$, because $5^2 = 25$.
- To undo a base-e exponential, isolate e^{rt} first, then take $\ln$ of both sides — $\ln$ and e cancel, leaving rt on its own to divide out.

becomes $\log x - \log y$ — subtract the logs, do not divide them.

- The decay constant is $k = \dfrac{\ln 2}{h} \approx \dfrac{0.693}{h}$, not $\frac{0.5}{h}$. A longer half-life gives a smaller k; the natural log is essential.
- For a compound-growth "how many whole years until it first exceeds a target", $n = \dfrac{\log(T/P)}{\log(1+i)}$ — then round UP to the next whole year, never to 3 s.f. With a cooling model, forgetting to subtract R before taking logs is the other classic slip.
- A difference on a log scale is a ratio in reality, not a sum: $+2$ on the Richter scale is $\times 100$ amplitude, and $+10$ dB is $\times 10$ intensity. Never drop the I_0 or the factor of 10 in the decibel formula.