PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Differentiation

SL 5.1–5.4 · AHL 5.6–5.8

ibtuition.sg · free

FORMULAS YOU MUST KNOW

$f'(x) = \frac{dy}{dx}$ — the gradient of $y = f(x)$ at each value of x

Gradient function:

$\frac{dy}{dx}$ — how fast y changes per unit increase in x

Rate of change:

$v = \frac{ds}{dt}$, $a = \frac{dv}{dt} = \frac{d^2s}{dt^2}$ — differentiate displacement for velocity, again for acceleration

Kinematics link:

$\frac{d}{dx}(ax^n) = an\,x^{n-1}$

Power rule:

$\frac{d}{dx}(c) = 0$ — a constant has zero gradient

Constant:

differentiate a polynomial term by term

Sum rule:

$\frac{d}{dx}(f(x))\Big|_{x=p} = f'(p)$ — the calculator's numerical-derivative template

Value at a point:

plot $y = f(x)$, then use the analyse/derivative ($\frac{dy}{dx}$) tool at $x = p$

From a graph:

$m = f'(p)$

Tangent gradient:

$y - f(p) = m(x - p)$ — its y-intercept is the constant c in $y = mx + c$

Tangent line:

$-\frac{1}{f'(p)}$ — the negative reciprocal (perpendicular to the tangent)

Normal gradient:

$y - f(p) = -\frac{1}{f'(p)}(x - p)$

Normal line:

KEY METHODS

- Rate of change is where AI meets the real world: $\frac{dP}{dt}$ is a growth rate, $\frac{dC}{dx}$ a marginal cost, $\frac{ds}{dt}$ a velocity. See Kinematics for the motion version in full.
- Before differentiating, rewrite every root and fraction as a power: $\sqrt{x} = x^{1/2}$ and $\frac{1}{x^2} = x^{-2}$. Then the power rule applies mechanically.
- To find where a graph is steepest, or to confirm a turning point, graph the model and read $\frac{dy}{dx}$ directly — the GDC returns $f'(p)$ to full accuracy in one step.

EXAM TRAPS TO AVOID

- Bring the power down first, then subtract one from the exponent. Reducing the power before multiplying, or forgetting that a lone constant differentiates to 0, are the classic slips.
- The numerical tool returns the gradient's value at one point, not the function $f'(x)$. When a question asks for $f'(x)$ in terms of x, or wants an exact Paper 1 answer, you must differentiate by hand.
- The normal gradient is the negative reciprocal $-\frac{1}{f'(p)}$, not simply $-f'(p)$. If the tangent is

- Read the wording carefully: it may want just the gradient, the full line equation, or only the y-intercept. Find m and the point once, then answer exactly what is asked.
- A local maximum is where f' changes from $+$ to $-$; a local minimum is where it changes from $-$ to $+$. For a positive cubic the smaller root of $f'(x) = 0$ is the maximum, the larger the minimum.
- Concave up where $f''(x) > 0$, concave down where $f''(x) < 0$. Running differentiation backwards is anti-differentiation — integrate $\dfrac{dy}{dx}$ and add $+C$, which a boundary condition pins down (see Integration).
- Calculus of $\sin$ and $\cos$ assumes radians — set the GDC to radian mode before differentiating or graphing a trig model. The one sign to remember is that $\cos x$ differentiates to $-\sin x$.

horizontal ($f'(p) = 0$) the normal is the vertical line $x = p$.

- Once you have the stationary x, substitute it back into $f(x)$ — not $f'(x)$ — to get the height y of the turning point.
- If $f''(x) = 0$ at a stationary point the test is inconclusive — fall back on the sign of $f'(x)$ on either side. And $f''(x) = 0$ on its own is not a point of inflexion unless the concavity actually changes.
- The chain rule's most-dropped piece is the derivative of the inside: $\dfrac{d}{dx}(ax + b)^n$ carries a factor a. Multiply by the derivative of the inside every single time.
- Answer the actual question — the area, volume or cost — not just the value of x. Reject any root that makes a length zero or negative, and keep only values inside the model's domain.