PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Differential Equations

AHL 5.14–5.16

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FORMULAS YOU MUST KNOW

$\frac{dQ}{dt}$ is the phrase "the rate of change of Q" — this is always the left-hand side

Rate of change:

"rate is proportional to the amount present" $\Rightarrow \frac{dQ}{dt} = kQ$

Proportional to size:

"rate is proportional to how far Q is from A" $\Rightarrow \frac{dQ}{dt} = k(Q - A)$

Proportional to a gap:

separate the variables and integrate, or step forward with Euler $y_{n+1} = y_n + h\, f(x_n, y_n)$

Then solve it:

$\frac{dN}{dt} = kN$

Model:

$N = N_0 e^{kt}$, where N_0 is the initial value at $t = 0$

Solution:

$k > 0$ grows, $k < 0$ decays; the doubling time or half-life comes from $e^{kt} = 2$ or $\frac{1}{2}$

Growth vs decay:

a continuous rate of $r\%$ per period means $k = \frac{r}{100}$

Continuous rate:

using (t_1, N_1): $k = \frac{1}{t_1} \ln \frac{N_1}{N_0}$

From the start value:

$k = \frac{1}{t_2 - t_1} \ln \frac{N_2}{N_1}$

From any two points:

$\frac{d\theta}{dt} = -k(\theta - \theta_s)$, where θ_s is the surrounding (ambient) temperature

Model:

$\theta = \theta_s + (\theta_0 - \theta_s)e^{-kt}$ — the object cools toward θ_s

Solution:

KEY METHODS

- Pick the sign of k from the context: growth uses $+k$, while decay or cooling uses $-k$ so the quantity falls. The constant of proportionality k itself is found later, from the data.
- Write "rate $=$" on the left first, then translate only the words that come after "proportional to" into the right-hand side. Almost every AI DE is one of the two templates above.
- Work out the exponent kt first, then raise e to that power on the GDC in one step — it avoids rounding a small k too early. See Exponents & Logarithms for the log work behind it.

EXAM TRAPS TO AVOID

- For decay the ratio $\frac{N_1}{N_0}$ is less than 1, so its $\ln$ is negative and k comes out negative. Keep that minus sign — it is what makes the model decay.
- Get e^{-kt} completely on its own before taking logs — you cannot $\ln$ a sum such as $\theta_s + (\ldots)e^{-kt}$ term by term. Subtract θ_s first.
- The $+C$ appears the moment you integrate, not at the end. And when a square root makes y explicit (as in $y^2 = x^2 + K$), choose the sign that matches the initial condition — if $y_0 > 0$, take the positive root.
- Update y using the old x and y, then advance x — do it one step at a time. Feeding the new x into the

- Read k straight off the equation whenever the DE is already written as $\frac{dN}{dt} = kN$. If instead you are given data, find k first with the method in §3.
- Divide the two amounts first, take $\ln$ of that single number, then divide by the elapsed time — three clean calculator steps.
- Keep the ambient term θ_s separate: only the difference $(\theta_0 - \theta_s)$ is multiplied by e^{-kt}. Once k is known you can predict any later temperature or the time to reach it.
- Substitute the given boundary point straight into the integrated equation to pin down the constant — do this before rearranging into an explicit y.

same step is the classic Euler error.