PHOTON ACADEMY · IB MATH AI HL CHEATSHEET

Complex Numbers

AHL 1.12–1.14
Argand · polar · De Moivre
phasors · eigenvalues

FORMULAS YOU MUST KNOW

$z = a + bi, \quad i^2 = -1$ rectangular form	$\lvert z\rvert = \sqrt{a^2+b^2}$ modulus (distance)
$\arg z = \tan^{-1}\frac{b}{a}$ angle, fix by quadrant	$z = r(\cos\theta + i\sin\theta) = re^{i\theta}$ polar / Euler
$(a+bi)(c+di) = (ac-bd) + (ad+bc)i$ multiply	$\dfrac{1}{c+di} = \dfrac{c-di}{c^2+d^2}$ divide by conjugate
$z^n = r^n(\cos n\theta + i\sin n\theta)$ De Moivre	$Z = R + Xi,\ \lvert Z\rvert = \sqrt{R^2+X^2}$ AC impedance

KEY METHODS

- **Modulus & argument:** plot $z = x + yi$ on the Argand diagram; $|z| = \sqrt{x^2+y^2}$, then read the angle and adjust for the quadrant so $-180^\circ < \arg z \le 180^\circ$.
- **Multiply:** expand and apply $i^2 = -1$ — the real part $ac - bd$ is a *subtraction*.
- **Divide:** multiply top and bottom by the conjugate; the denominator becomes the real number $c^2 + d^2$.
- **Powers (De Moivre):** raise the modulus to the power *and* multiply the angle: $|z^n| = |z|^n$.
- **AC phasors:** write each $A\angle\phi$ as $A\cos\phi + iA\sin\phi$, add in rectangular form, then take the modulus for the resultant amplitude.
- **Complex eigenvalues (2×2):** solve $\lambda^2 - (\mathrm{tr})\lambda + \det = 0$; a negative discriminant gives $\lambda = \alpha \pm \beta i$ → the system spirals with period $T = \frac{2\pi}{\beta}$.

EXAM TRAPS TO AVOID

- Signs of a, b never matter for $|z|$ — you square them.
- Always **sketch z first** so the argument lands in the right quadrant.
- The real part of a product is $ac - bd$ (from $i^2 = -1$) — don't drop the $-bd$ term or its sign.
- Conjugate of $c + di$ is $c - di$; the new denominator is $c^2 + d^2$, not $c^2 - d^2$.
- Powers scale the modulus *and* the angle — it's $|z|^n$, never $n|z|$.
- Out-of-phase AC amplitudes do **not** add arithmetically — always go via rectangular form.
- A spiral's **period uses the imaginary part only**: $T = \frac{2\pi}{\beta}$; the real part sets growth/decay.