Approximation & Error

FORMULAS YOU MUST KNOW

keep n digits after the decimal point; the very next digit decides — ≥ 5 rounds up, < 5 rounds down
Round to n d.p.:

$5.6449 \to 5.64$ (2 d.p.), because the third decimal is 4
Example:

the first non-zero digit; in 0.00408 the first significant figure is the 4
First s.f.:

keep n significant digits, use the next digit to round, then fill remaining whole-number places with zeros
Round to n s.f.:

for any value between 1 and 10, quoting 3 s.f. is identical to 2 d.p. — e.g. $6.7194 \to 6.72$
Handy identity:

$a \times 10^k$ with $1 \leq a < 10$ and k an integer
Form:

k counts the places the decimal point moves; large numbers give $k > 0$, numbers below 1 give $k < 0$
Sign of k:

$6\,720\,000 = 6.72 \times 10^6$; $0.00048 = 4.8 \times 10^{-4}$
Examples:

v_A = approximate / measured value; v_E = exact / accepted value
Symbols:

a value rounded to a unit u could really be anything within $\pm\frac{u}{2}$
Half-unit rule:

$v_A-\tfrac{u}{2}\le v_{\text{true}}Interval:

nearest $0.1 \Rightarrow \pm 0.05$; nearest whole cm $\Rightarrow \pm 0.5$; nearest 10 g $\Rightarrow \pm 5$ g
Examples:

KEY METHODS

- On a GDC, keep the full stored value through every intermediate step and round only the final answer. Rounding halfway and then reusing the rounded number is the classic way to lose an accuracy mark.
- Fix the last kept digit by rounding with the next figure, never by chopping the number off. 6.798 to 3 s.f. is 6.80, not 6.79.
- Move the point until exactly one non-zero digit sits in front of it, then count the moves for k: point moved left $\Rightarrow$ positive k; a small number (point moved right) $\Rightarrow$ negative k.

EXAM TRAPS TO AVOID

- Round in one step from the exact value using the single deciding digit. Do not chain-round from the far right ($5.6449 \to 5.645 \to 5.65$ is wrong — the deciding digit is the 4, so it is 5.64).
- Rounding to s.f. must preserve the size of the number. $48\,700$ to 2 s.f. is $49\,000$ — replace the dropped whole-number digits with zeros, do not delete them (writing 49 changes the number by a factor of a thousand).
- The coefficient must satisfy $1 \leq a < 10$. 72×10^5 is not standard form — adjust it to 7.2×10^6.

- The modulus makes percentage error always positive, so it does not matter whether your approximation is an over- or under-estimate — you report the size of the gap.
- Add the half-unit for the upper bound, subtract it for the lower bound. The true value can equal the lower bound but stays strictly below the upper bound.
- Get each side's upper and lower bound first, then combine — never round back to the given figures partway through, or the compounded error disappears.
- A difference flips one bound: for $a-b$ the maximum uses a's upper bound and b's lower bound, $(a-b)_{\max}=a_{\max}-b_{\min}$. Any quantity you subtract contributes its opposite bound.

Significant figures of a small number start at the first non-zero digit, not at the leading zeros.

- Always divide by the EXACT (accepted) value — it sits in the denominator, never the approximation. Dividing by the measured value instead is the single most common slip, and never forget the $\times 100$.
- The unit u is the last place quoted. A length given as 1.5 m (1 d.p.) has $u=0.1$, so the swing is ± 0.05 — not ± 0.5. Read the precision off the last digit, not the first.
- To make a fraction large, make the bottom small. Feeding both quantities' upper bounds into a quotient is wrong — the denominator's bounds swap, so the maximum density uses $m_{\max}$ over $V_{\min}$.
- The model's prediction is the approximation; the measured value is treated as exact, so it goes in the denominator. Evaluate b^t with the full power t, not $t-1$.